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156. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Find the volume common to the two solids $x^2+y^2+z^2=a^2$ and $xz^2=(a-x)(x^2+y^2)$.

I. Solution by the PROPOSER.

The limits of z are $z=\sqrt{\dfrac{a-x}{x}(x^2+y^2)}$ to $z=\sqrt{(a^2-x^2-y^2)}$.

Eliminating z, $y=\sqrt{[x(a-x)]}$.

$\therefore$ The limits of y are 0 and $\sqrt{[x(a-x)]}=y'$; the limits of x are 0 and a.

$$\therefore\ V=4\int_0^a\int_0^{y'}\left[\sqrt{(a^2-x^2-y^2)}-\sqrt{\frac{a-x}{x}(x^2+y^2)}\right]dxdy,$$

$$=2\int_0^a\left[(a^2-x^2)\sin^{-1}\sqrt{\frac{x}{a+x}}-x\sqrt{(ax-x^2)}\log\left(\frac{\sqrt{a}+\sqrt{(a-x)}}{\sqrt{x}}\right)\right]dx.$$

Let $x=a\tan^2\theta$ in the first term, and $x=a\sin^2\theta$ in the second term.

$$\therefore\ V=4a^3\int_0^{\frac{1}{4}\pi}\theta(1-\tan^4\theta)\tan\theta\sec^2\theta d\theta-4a^3\int_0^{\frac{1}{2}\pi}\sin^4\theta\cos^2\theta\log\left(\frac{1+\cos\theta}{\sin\theta}\right)d\theta$$

$$=\tfrac{2}{3}\pi a^3-\tfrac{5}{4}a^3+\tfrac{1}{4}a^3\int_0^{\frac{1}{2}\pi}\log(\tan\tfrac{1}{2}\theta)d\theta.$$

$$\int_0^{\frac{1}{2}\pi}\log(\tan\tfrac{1}{2}\theta)d\theta=\tfrac{1}{2}\int_0^{\frac{1}{2}\pi}\log\left(\frac{1-\cos\theta}{1+\cos\theta}\right)d\theta=-\int_0^{\frac{1}{2}\pi}(\cos\theta+\tfrac{1}{3}\cos^3\theta+\tfrac{1}{5}\cos^5\theta+\ldots..)d\theta$$

$$=-2\int_0^{\frac{1}{2}\pi}(\cos\theta+\tfrac{1}{3}\cos3\theta+\tfrac{1}{5}\cos5\theta+\ldots.)d\theta=-2(1-1/3^2+1/5^2-1/7^2+1/9^2-\ldots)$$

$$=-16\left[\frac{1}{1^2.3^2}+\frac{3}{5^2.7^2}+\ldots+\frac{2n-1}{(4n-3)^2(4n-1)^2}\right]$$

$=-1.832$ nearly $=-16[.114488335]$.

$\therefore\ V=\tfrac{2}{3}\pi a^3-\tfrac{5}{4}a^3-.458a^3=\tfrac{2}{3}\pi a^3-1.708a^3$.

II. Solution by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Transforming the two surfaces to polar coördinates, the equations of the first are $\rho=a$, and of the second, $\rho=a\sin\phi\sec\theta$. The equations of the intersection of the two surfaces are $\left.\begin{matrix}\rho=a\\ \sin\phi=\cos\theta\end{matrix}\right\}$.

The volume common to the two solids bounded by the surfaces is

$$V=4\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi-\phi}\int_{a\sin\phi\sec\theta}^{\frac{1}{2}\pi}\rho^2\sin\phi d\phi d\theta d\rho=\tfrac{4}{3}a^3\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi-\phi}(1-\sin^3\phi\sec^3\theta)\sin\phi d\phi d\theta,$$

$$=\tfrac{4}{3}a^3[\tfrac{1}{2}\pi-1\tfrac{5}{6}+\tfrac{3}{16}\int_0^{\frac{1}{2}\pi}\log\tan\frac{\phi}{2}d\phi=\tfrac{2}{3}\pi a^3-\tfrac{5}{4}a^3+\tfrac{1}{4}a^3\int_0^{\frac{1}{2}\pi}\log\tan\frac{\phi}{2}d\phi],$$

$$=\tfrac{2}{3}\pi a^3-\tfrac{5}{4}a^3-\tfrac{1}{4}a^3\sum_{n=1}^{n=\infty}\frac{2n-1}{[4n-3]^2\,[4n-1]^2}=.386a^3\text{, nearly.}$$

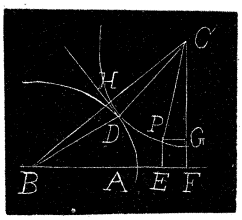
On the integration of $\int_0^{\frac{1}{2}\pi}\log\tan\frac{\phi}{2}d\phi$, see the remarks on Prize Problem, No. 123, Calculus.

157. Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

Two equal ellipses are tangent to each other at the vertices of the major axes. If one of them be rolled on the other; find (1) the equation and area of the curve described by the vertex, and (2) by the center.

Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and the PROPOSER.

Let a and b be the semi-axes of both ellipses; B the center of the fixed ellipse: C, the center of the rolling ellipse; P, its vertex; and D, the point of contact; $e^2=(a^2-b^2)/a^2$. Let $BE=x$, $BF=m$, $PE=y$, $CF=n$, and $\angle ABC=\angle PCB=\theta$.



Then, $BC=2a\sqrt{(1-e^2\sin^2\theta)}$=twice the length of the perpendicular from B on the tangent at D.

$m=2a\cos\theta\sqrt{(1-e^2\sin^2\theta)}$, $n=2a\sin\theta\sqrt{(1-e^2\sin^2\theta)}$,

$x=m-PG=2a\cos\theta\sqrt{(1-e^2\sin^2\theta)}-a\cos2\theta$,

$y=n-CG=2a\sin\theta\sqrt{(1-e^2\sin^2\theta)}-a\sin2\theta$.

$$x^2+y^2=r^2=a^2+4a^2(1-e^2\sin^2\theta)-4a^2\cos\theta\sqrt{(1-e^2\sin^2\theta)},$$

the equation of the locus of the vertex.

$$\text{Area}=a^2\int_0^{\pi}[5-4e^2\sin^2\theta-4\cos\theta\sqrt{(1-e^2\sin^2\theta)}]d\theta=\pi a^2(5-2e^2)=\pi(3a^2+2b^2).$$

$BC^2=\rho^2=4a^2(1-e^2\sin^2\theta)$, the equation of the locus of the center.

$$\text{Area}=4a^2\int_0^{\pi}(1-e^2\sin^2\theta)d\theta=2\pi a^2(2-e^2)=2\pi(a^2+b^2).$$

Also solved by *J. SCHEFFER*, and *G. W. GREENWOOD*.

MECHANICS.

147. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

A particle mass m is attached to one end of a string, the other end of which is fixed. It is projected horizontally with such a velocity that it would rise to a position in which